

Research article

# COMPUTER – AIDED TORSIONAL ANALYSIS OF DETERMINATE BOX CULVERTS

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## ABSTRACT

Saint – Venant’s theory of torsion has been universally employed in the analysis of box culverts. This simplified approach of torsional analysis of structures neglected the effects of restrained warping. However, another approach based on Vlasov’s theory considered and induced warping of thin walled structures. The main disadvantage of the theory is the complexity of its equation. And so, the method is hardly used in torsional analysis of structures. In this work, a computer program based on equations derived from Vlasov’s theory, were developed for easy, quick and accurate torsional analysis of box culverts. In addition, computer program based on De Saint venant’s theory of torsion was developed for torsional analysis of box culverts.

The results obtained from the computer programs for the torsional analysis of box culverts compare favourably with corresponding results obtained from manual torsional analysis.

**Keywords:** Computer program, torsional analysis, Vlasov theory, warping, determinate box culverts.

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## INTRODUCTION

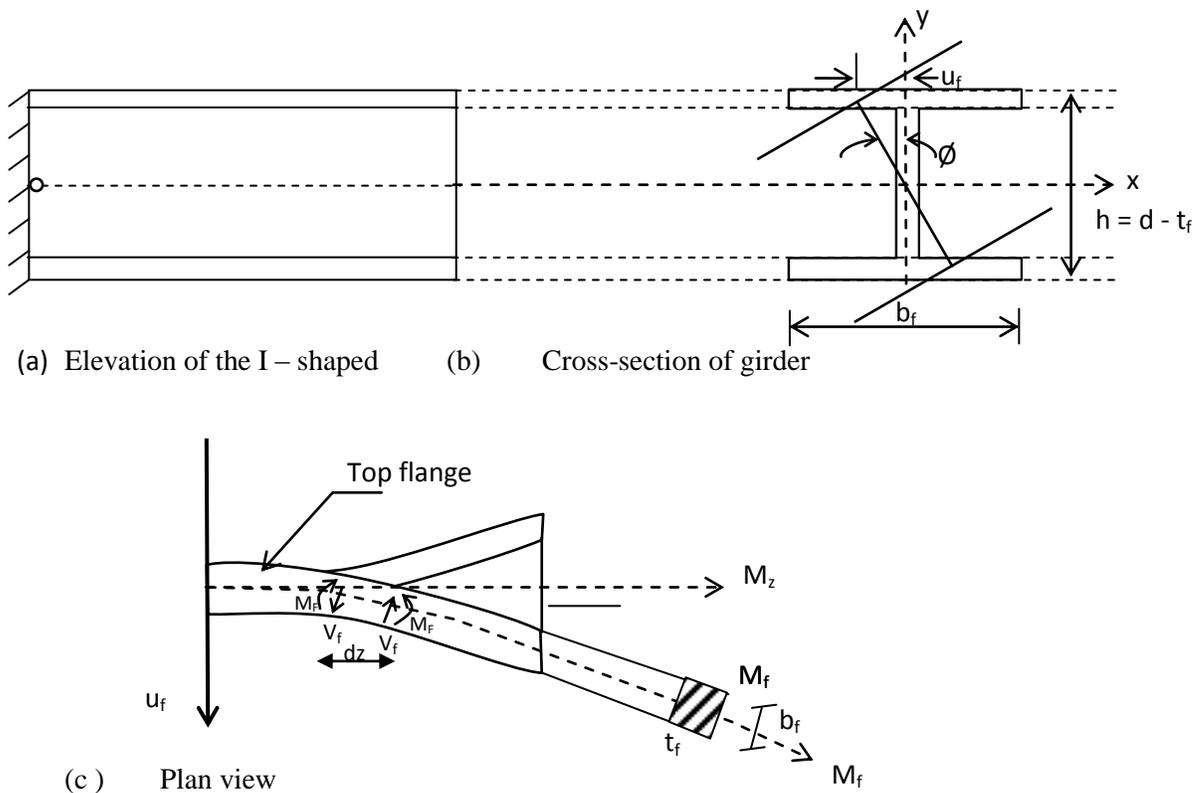
Culverts are very important bridge structure that used for conducting streams or drains under embankments. They can either be circular, rectangular (i.e. box) or elliptical in shape. The use of box culverts in modern system is dominant due to their economy. Recently, thin-walled members of open sections such as I-section, channel section, are frequently employed in the design and construction of culverts due to its economy. When these open sections are used, their torsional resistances are increased by creating a box shape i.e. by welding one or two side plates between the flanges of an I-section or a channel-section for a portion of its length (AISC, 2003).

However, box culverts in general, have the shortcoming that their cross-section distort under eccentric loading and warp out of plane section. According to Heins (1975), the original cross-sectional plane becomes a warped surface. In addition, the opening in the box culvert may reduce the strength of its section, if it is not properly analyzed and design. Hence, it is essential to study the warping behavior and the overall torsional behavior of box culverts with different support conditions.

There are few studies available on thin walled members with such cross sectional distortion. One of such studies is based on De Saint-Venant theory of torsion which assumed free warping of member section under twist (Meijers, 1998). This simplified approach has been universally used in the conventional methods of studying the warping and overall behavior of box culverts. But, another study based on torsion theory of Vlasov (1959) incorporated the effects of restrained warping. In this work, Vlasov's torsional theory is used to analyze box culverts and the results obtained are compared with results obtained from De-Saint-Venant method of analysis.

## 2.0 THEORETICAL BACKGROUND

The program developed for torsional analysis of box culverts is based on Vlasov's theory. This theory incorporated warping in the formulation of its differential equation for angle of twist,  $\phi$ . In formulating the differential equation, Vlasov considered a thin – walled I-section with applied moment,  $M_z$ , as shown in figure 1. Assuming that the web of the I-section is not deformed, the applied moment,  $M_z$ , will be resisted by the flanges. And the shear deformations are caused by cross-bending as shown in fig 1.



**Figure1:** Warping of an I-shaped girder

The differential equation for the angle of twist,  $\phi$ , as developed by Vlasov is given by Eqn (1).

$$M_z = M_z^{ST} + M_z^W \quad (1)$$

Where  $M_z^{ST}$  = Saint – Venant pure torsional moment.

$M_z^W$  = warping torsion moment.

But, Saint – Venant gave pure torsional moment,  $M_z^{ST}$  as

$$M_z^{ST} = GJ\phi' \quad (2)$$

And warping torsion moment,  $M_z^W$ , as derived by Vlasov is given by Eqn (3).

$$M_z^W = -EC_w\phi''' \quad (3)$$

Where  $G$  = shear modulus of the material

$J$  = torsion constant of the member's cross – section

$\phi$  = angle of twist

$\phi'$  = First derivative of angle of  $\phi$  with respect to  $Z$ .

$\phi'''$  = Third derivative of angle of twist,  $\phi$  with respect of  $Z$ .

$E$  = Elastic modulus of the material of the element.

$C_w$  = Warping of the material's cross-section

When Eqns (2) and (3) are substituted into Eqn (1), it becomes:

$$M_z = GJ\phi' - EC_w\phi''' \quad (4)$$

$$\text{But, } Bi = -EC_w \frac{d^2\phi}{dz^2} = -EC_w\phi'' \quad (5)$$

Where  $Bi$  = Bi – moment

Differentiating  $Bi$  with respect to  $Z$  yields:

$$\frac{\partial Bi}{\partial z} = -EC_w\phi''' = -EC_w \frac{d^3\phi}{dz^3} \quad (6)$$

$$\text{Therefore, } M_z = GJ\phi' + \frac{\partial Bi}{\partial z} \quad (7)$$

Eqns (4) and (7) are the Vlasov's differential equations for computing the torque,  $M_z$ , on a box culvert.

The general solution of eqn (4) is given in the following hyperbolic form:

$$\phi = A + BCosh\beta z + CSinh\beta z + \frac{M_z z}{\beta EC_w} \quad (8)$$

Where  $A$ ,  $B$  and  $C$  are constants of integration, that can be obtained using boundary conditions.

### 3.0 SOLUTION OF VLASOV'S DIFFERENTIAL EQUATION FOR DETERMINATE BOX CULVERTS/GIRDERS.

#### 3.1 Case 1: Cantilever Box Culvert/Girder

Consider a determinate cantilever girder subject to a concentrated torque,  $M_z$ , applied at a distance,  $\alpha L$ , from the left end of the girder shown in fig 2.

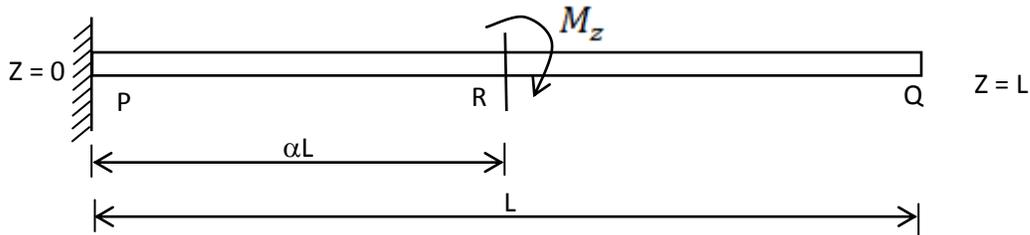


Figure 2: Cantilever girder box culvert subject to a torque,  $M_z$ , at point R

The boundary conditions at the free end of the cantilever girder/culvert are as follows:

- (a) For free warping condition to exist.
  - (i)  $\mathbf{Bi} = \phi$
  - (ii)  $\phi'' = 0$
- (b) For no warping shear condition to exist
  - $\phi''' = 0$

The solution to Vlasov's differential equation i.e Eqn (8), was used in deriving the angle of twist,  $\phi$ , for a cantilever girder.

For the region lying between points P and R, i.e.  $0 < Z < \alpha L$

$$\phi = \frac{M_z}{\beta GJ} [Z\beta - \text{Sinh}\beta Z + \text{Cosh}\beta z (\text{Coth}(\beta\alpha L) - \text{Cosech}(\beta\alpha L)) + (\text{Cosech}(\beta\alpha L) - \text{Coth}(\beta\alpha L))] \quad (9)$$

Differentiating the angle of twist,  $\phi$ , once, twice and thrice with respect to Z, yields Eqns (10), (11) and (12) respectively:

$$\phi' = \frac{M_z}{GJ} [1 - \text{Cosh}\beta z + \text{Sinh}\beta z (\text{Coth}(\beta\alpha L) - \text{Cosech}(\beta\alpha L))] \quad (10)$$

$$\phi'' = \frac{\beta M_z}{GJ} [\text{Cosh}\beta z (\text{Coth}(\beta\alpha L) - \text{Cosech}(\beta\alpha L)) - \text{Sinh}\beta z] \quad (11)$$

$$\phi''' = \frac{\beta^2 M_z}{GJ} [\text{Sinh}\beta z (\text{Coth}(\beta\alpha L) - \text{Cosech}(\beta\alpha L)) - \text{Cosh}\beta z] \quad (12)$$

For the region lying between R and Q i.e.  $(\alpha L < Z < L)$ :

$$\phi = \frac{M_z}{\beta GJ} [\beta z + H_1 \text{Sinh}\beta z - H_1 \text{Tanh}\beta L \text{Cosh}\beta z + \text{Cosech}(\beta\alpha L) - \text{Coth}(\beta\alpha L)] \quad (13)$$

Also differentiating the angle of twist,  $\phi$  (i.e Eqn (13) with respect to Z, gives Eqns (14), (15) and (16) respectively

$$\phi' = \frac{M_z}{GJ} [1 + H_1 \text{Cosh}\beta z - H_1 \text{Tanh}\beta L \text{Sinh}\beta z] \quad (14)$$

$$\phi'' = \frac{M_z \beta H_1}{GJ} [\text{Sinh}\beta z - \text{Tanh}\beta L \text{Cosh}\beta z] \quad (15)$$

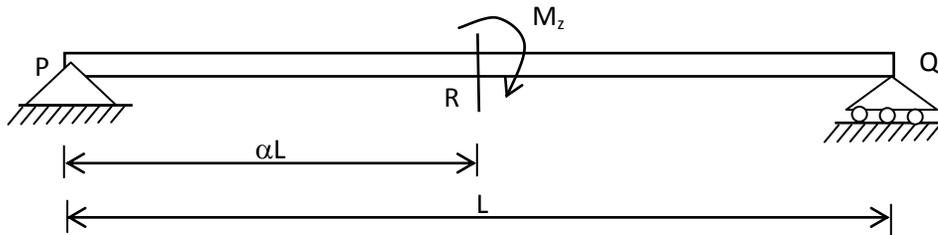
$$\phi''' = \frac{M_z \beta^2 H_1}{GJ} [\text{Cosh}\beta z - \text{Tanh}\beta L \text{Sinh}\beta z] \quad (16)$$

Where

$$H_1 = \left[ \frac{\text{Coth}(\beta\alpha L) - \text{Cosech}(\beta\alpha L) - \text{Tanh}(\beta\alpha L)}{\text{Tanh}(\beta\alpha L) - \text{Tanh}\beta L} \right]$$

### 3.2 Case 2: Simply Supported Box Culvert/Girder

This is another form of determinate box culvert. Consider a simply support box culvert subjected to a concentrated torque,  $M_z$ , at a distance ' $\alpha L$ ' from the left support (see fig 3)



**Figure 3:** Simply supported Girder /box culvert subject to a torque,  $M_z$ , at point R

The boundary conditions at the pinned end of the simply supported beam, are given below

- (i) For no twist to exist,  $\phi = 0$
- (ii) For free warping condition to exist,  $\mathbf{Bi} = 0$

The application of these boundary conditions into vlasov's differential equation i.e Eqn (8) yielded the following equations:

For the region lying between points P and R, i.e. ( $0 < Z < \alpha L$ ).

The equation of the angle of twist,  $\phi$  is given by:

$$\phi = \frac{M_z}{\beta GJ} \left[ Z\beta - \frac{\text{Sinh}\beta Z}{\text{Cosh}(\beta\alpha L)} \right] \quad (17)$$

The first,  $\phi'$ , second,  $\phi''$  and third,  $\phi'''$  derivatives are as follows:

$$\phi' = \frac{M_z}{GJ} \left[ 1 - \frac{\text{Cosh}\beta Z}{\text{Cosh}(\beta\alpha L)} \right] \quad (18)$$

$$\phi'' = -\frac{\beta M_z}{GJ} * \frac{\text{Sinh}\beta Z}{\text{Cosh}(\beta\alpha L)} \quad (19)$$

$$\phi''' = -\frac{\beta^2 M_z}{GJ} * \frac{\text{Cosh}\beta Z}{\text{Cosh}(\beta\alpha L)} \quad (20)$$

For the region lying between R and Q (i.e  $\alpha L < Z < L$ )

$$\phi = \frac{M_z}{\beta GJ} \left\{ (Z - L)\beta + \left[ \frac{\text{Sech}(\beta\alpha L) \text{Tanh}\beta L \text{Cosh}\beta z}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} \right] - \left[ \frac{\text{Sech}(\beta\alpha L) \text{Sinh}\beta z}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} \right] \right\} \quad (21)$$

And differentiating the angle of twist,  $\phi$  once, twice and thrice with respect to Z, gives Eqns (22), (23) and (24) respectively.

$$\phi' = \frac{M_z}{GJ} \left[ 1 + \frac{\text{Sech}(\beta\alpha L) \text{Sinh}\beta z \text{Tanh}\beta L}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} - \frac{\text{Sech}(\beta\alpha L) \text{Cosh}\beta z}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} \right] \quad (22)$$

$$\phi'' = \frac{\beta M_z}{GJ} \left[ \frac{\text{Sech}(\beta\alpha L) \text{Cosh}\beta z \text{Tanh}\beta L}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} - \frac{\text{Sech}(\beta\alpha L) \text{Sinh}\beta z}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} \right] \quad (23)$$

$$\phi''' = \frac{\beta^2 M_z}{GJ} \left[ \frac{\text{Sech}(\beta\alpha L) \text{Sinh}\beta z \text{Tanh}\beta L}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} - \frac{\text{Sech}(\beta\alpha L) \text{Cosh}\beta z}{[1 - \text{Tanh}\beta L \text{Coth}(\beta\alpha L)]} \right] \quad (24)$$

### 3.3 Vlasov's Torsional Stress, $\sigma_t$

(a) **De Saint – Venant's Equation For Angle of Twists,  $\phi$ :** In his own torsion theory, De Saint – Venant, who assumed free warping, derived the equation for angle of twist,  $\phi_D$ , as given by:

$$\phi_D = \left[ \frac{M_z Z}{GJ} \right]_{Z=0,L} \quad (25)$$

(b) **De Saint – Venant's Stress,  $\tau_t$**

The following torsional shear stress,  $\tau_t$ , equation for a box culvert, was derived by De Saint – Venant.

$$\tau_t = G t \phi_D' \quad (26)$$

Where  $\phi_D' =$  the first derivative of angle twist with respect to Z =  $\frac{M_z}{GJ}$

G = shear modulus, and

t = thickness of the box culvert

(c) **Vlasov's Stresses,  $\sigma_z$**

Vlasov's torsional stress,  $\sigma_z$ , is made of three parts, namely, shear stress due to rotation of member,  $\tau_t$ , shear stress due to restrained warping,  $\tau_w$  and axial normal warping stress,  $\sigma_w$ . And so, the equation for torsional stress as derived by Vlasov is as follows:

$$\sigma_z = \tau_t + \tau_w + \sigma_w \quad (27)$$

Where  $\sigma_w = \frac{B_i}{C_w} W_n = -E W_n \phi''$

$$\tau_w = -\frac{E S_w \phi'''}{t}$$

$W_n$  = the normalized unit warping of the cross – section of the culvert

$S_w$  = warping statical moment of the selected section of the culvert

### 3.4 Computer Program

First and foremost, the algorithm of the computer program was developed. This was followed by the development of the flowchart of the program. Finally the computer programs were designed and used for torsional analysis of box culverts.

#### 3.4.1 Algorithm for the Computer Program

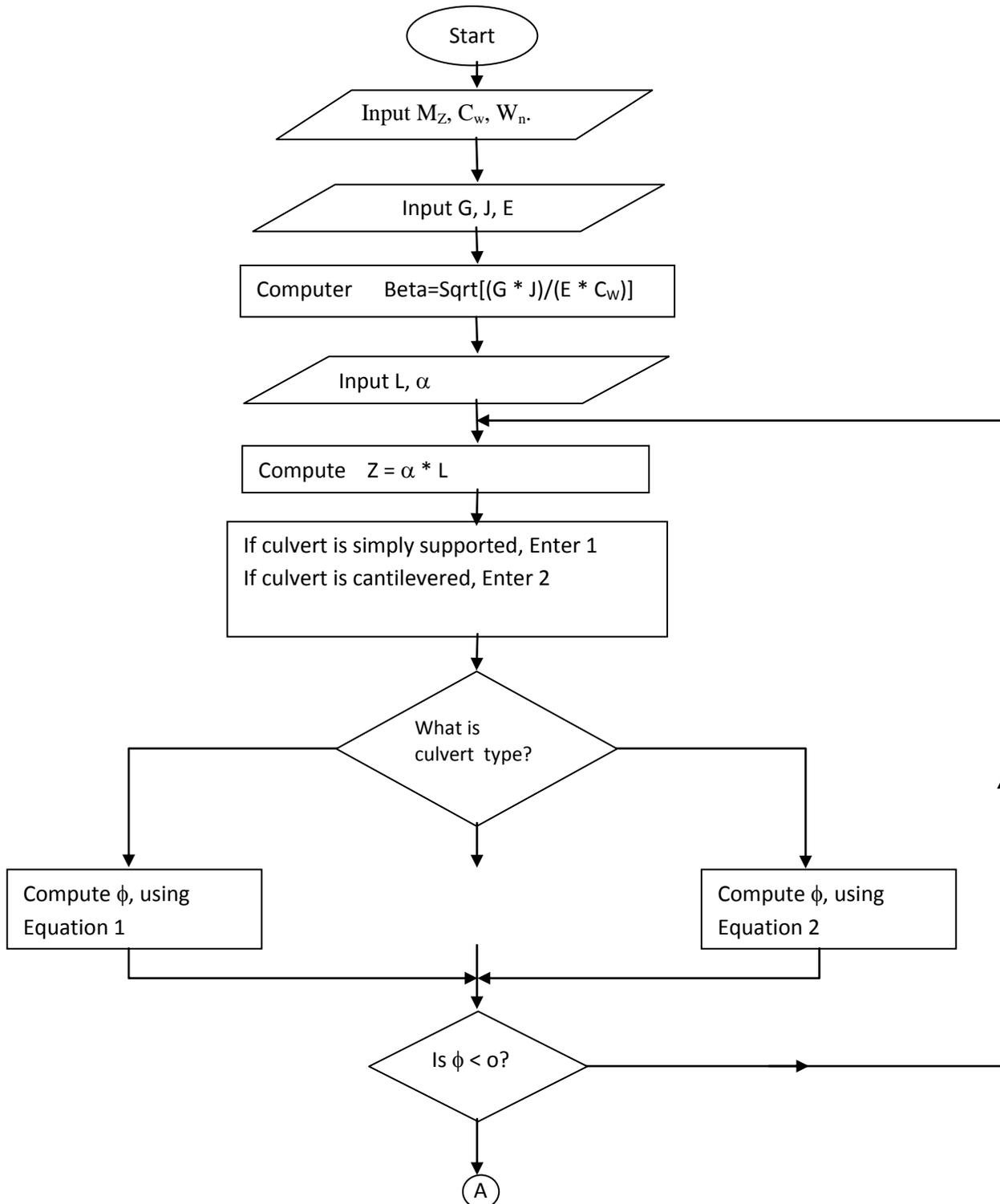
The following is the algorithm of the computer program used in Torsional analysis of Box culvert using Vlasov and De-Saint Venant torsional theories, and comparison of the two theories:

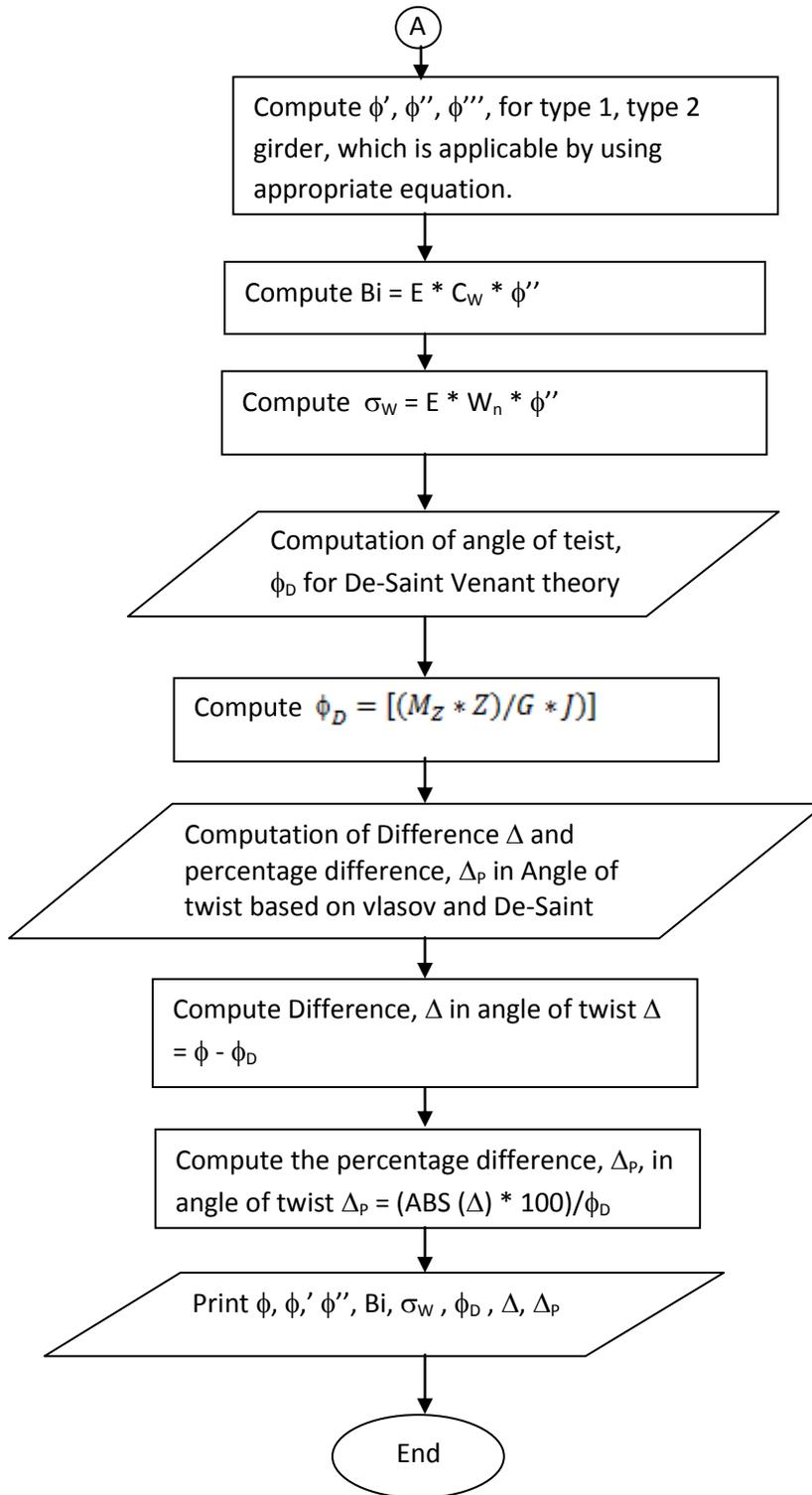
- (i) Input the torsional moment,  $M_z$ , warping constant,  $C_w$  and normalized unit warping,  $W_n$ .
- (ii) Input the material properties, i.e. shear modulus,  $G$ , Elastic modulus,  $E$ , and torsional constant,  $J$ , of the cross-section of the box culvert.
- (iii) Computer beta,  $\beta = \sqrt{\frac{GJ}{EC_w}}$
- (iv) Input the span,  $L$ , and the fraction,  $\alpha$ , of the length at which the torsional moment is applied.
- (v) Compute  $Z = \alpha L$
- (vi) Define different types of the Culvert
- (vii) Enter 'type 1' for simply supported box culvert
- (viii) Enter 'type 2' for cantilevered box culvert
- (ix) Is the girder type 1 or type 2?
- (x) If type 1, use equation (1) to computer the angle of twist,  $\phi$
- (xi) If type 2, use equation (2) to compute the angle of twist,  $\phi$
- (xii) Is  $\phi < 0$ ? Go to Step v
- (xiii) Otherwise compute  $\phi'$ ,  $\phi''$  using suitable equations
- (xiv) Compute  $B_i = -EC_w \phi''$
- (xv) Compute  $\sigma_w = -E W_n \phi''$
- (xvi) Compute  $\phi$  based on De-Saint Venant theory using  $\phi_D = \left[ \frac{M_z Z}{GJ} \right]_{z=0,L}$
- (xvii) Compute difference of the angles of twist of Vlasov and De-Saint Venant using  $\Delta = \phi - \phi_D$
- (xviii) Compute the percentage difference of angle of twist based on vlasov using  $\Delta_p = \left[ \frac{\Delta}{\phi_D} \right] * 100$
- (xix) Print  $\phi$ ,  $\phi'$ ,  $\phi''$ ,  $B_i$ ,  $\sigma_w$ ,  $\phi_D$ ,  $\Delta$ ,  $\Delta_p$
- (xx) End

Note: equations (1) and (2) shall be provided during the running of the program. (See eqn (17) to eqn (24))

### 3.4.2 Flow Chart

The flow chart for the computer program used in Torsional Analysis of Box Culvert is as illustrated in fig.4.





**Figure 4:** Flow chart for the computer program used in Torsional Analysis of Box Culvert

### 3.4.3 Program Design

Program written in Visual Basic 6.0 was designed for the torsional analysis of box culverts based on Vlasov's and Saint – Venant's theories of torsion. This program is presented in the appendix.

### 3.4.4 Executed Program

The designed programs were executed for simply supported and cantilever box culverts subject to various torsional moments,  $M_z$ , of values  $3.0 \times 10^4$  kN.m,  $3.50 \times 10^4$  kN.m,  $4.0 \times 10^4$  kN.m,  $4.5 \times 10^4$  kN.m,  $5.0 \times 10^4$  kN.m, and  $6.0 \times 10^4$  kNm applied successively at their mid-spans. The lengths of each type of box culvert, is 60m, and the properties of the box culverts are as follows:

$C_w = 39.44m^6$ ,  $G = 1.3043826 \times 10^7$  KN/m<sup>2</sup> and  $W_n = 5.1182m^2$ ,  $E = 3.0 \times 10^6$  KN/m<sup>2</sup>,  $J = 20.62m^4$  and  $Sw = 2.824m^4$ .

The executed programs are presented below.

## 4.0 RESULTS AND ANALYSIS

The results of the manual analysis and computer analysis based on De Saint Venant's torsion theory are given and compared in table 1. And the manual and computer results based on Vlasov's theory are presented in Table 2.

**Table 1:** Saint Venant's Manual and Computer Results

Box Culvert Type	Applied Torque, $M_z \times 10^5$ (Nm)	Angle of Twist, $\phi$ ( $10^{-5}$ rad.)		Absolute % Difference
		Manual results	Computer results	
Simply Supported Box Culvert	300	167.300	167.313	0.0078
	350	195.000	195.199	0.1000
	400	223.000	223.084	0.0377
	450	251.000	250.970	0.0119
	500	279.000	278.855	0.0520
	550	307.000	306.740	0.0848
	600	335.000	334.627	0.1115
Cantilever Box Culvert	300	334.600	334.627	0.0081
	350	390.000	390.398	0.1019
	400	446.000	446.169	0.0379
	450	502.000	501.940	0.0120
	500	558.000	557.711	0.0518
	550	614.000	613.482	0.0084
	600	670.000	669.253	0.1116

**Table 2:** Vlasov's Manual and Computer Results

Box Culvert Type	Applied Torque, $M_z \times 10^5$ (Nm)	Angle of Twist, $\phi$ ( $10^{-5}$ rad.)		Absolute % Difference
		Manual results	Computer results	
Simply Supported Box Culvert	300	156.000	155.615	0.2474
	350	182.000	181.551	0.2473
	400	207.000	207.476	0.2294
	450	233.000	233.423	0.1812
	500	259.000	259.359	0.1384
	550	285.000	285.295	0.0848
	600	311.000	311.231	0.1034
Cantilever Box Culvert	300	288.000	287.836	0.0057
	350	336.000	335.809	0.0057
	400	384.000	383.782	0.0057
	450	432.000	431.754	0.0057
	500	480.000	479.727	0.0057
	550	528.000	527.700	0.0057
	600	576.000	575.672	0.0057

#### 4.1 Comparison of Results

The results of the manual analysis and computer analysis based on De Saint – Venants theory are compared in table 4.1. The maximum percentage difference between the angles of twist,  $\phi$ , obtained from the manual analysis and computer analysis is 0.1116% , which is very negligible. And the comparison of manual results and computer results based Vlasov's results in table 4.2, yielded. The maximum percentage difference between the angles of twist,  $\phi$ , obtained from the manual analysis and computer analysis is 0.2474% , which is also negligible.

#### 5.0 CONCLUSION

The following conclusion can be drawn from this investigation:

- The comparison of the result obtained from conventional method (i.e. De Saint-Venant method) and Vlasov method showed that the deformation,  $\phi$  due to Saint-Venant is greater than that due to Vlasov.
- The values of angle of twist,  $\phi$  obtained from manual analysis for both theories are comparable to their computer results.

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## APPENDIX

### Computer program for analysing determinate thin – walled box culvert

```
Rem ONE COMPONENT
```

```
Cls
```

```
NN = InputBox("HOW TORQUES ARE YOU INTERESTED IN?"): NN = NN * 1
```

```
ReDim MZ(NN), PHY(NN), PHY1(NN), PHY11(NN), PHY111(NN), PHYV(NN), PD(NN), BiM(NN),  
TW(NN), WNS(NN)
```

```
Print " THE PROGRAM WAS WRITTEN BY"
```

```
Print: Print
```

```
Print " ONWUKA ANYAOGU"
```

```
Print:
```

```
WWWWW = InputBox("CLICK OK. TO CONTINUE"): Cls
```

```
ee = 2.718281828
```

```
J = InputBox("WHAT IS THE VALUE torsional constant OF 'J'in M4"): J = J * 1
```

```
For X = 1 To NN
```

```
MZ(X) = InputBox("WHAT IS THE VALUE OF applied torque 'MZ' in NM", X): MZ(X) = MZ(X) * 1
```

```
Next X
```

```
E = InputBox("WHAT IS THE VALUE OF Young's Modulus'E' in N/M2"): E = E * 1
```

```
CW = InputBox("WHAT IS THE VALUE OF warping constant 'CW' in M6"): CW = CW * 1
```

```
A = InputBox("WHAT IS THE VALUE OF cross sectional area 'A' in M2"): A = A * 1
```

```
'SW = InputBox("WHAT IS THE VALUE OF 'SW'"): SW = SW * 1
```

```
WN = InputBox("WHAT IS THE VALUE OF normalize warping constant 'WN'in M2"): WN = WN * 1
```

```
U = InputBox("WHAT IS THE VALUE poisson ratio OF 'U'"): U = U * 1
```

```
T = InputBox("WHAT IS THE VALUE OF 'THICKNESS OF BEAM'"): T = T * 1
```

```
G = E / 2 / (1 + U)
```

```
B = ((G * J) / (E * CW)) ^ 0.5
```

```
BEAM = InputBox("ENTER 1 FOR SIMPLE; 2 FOR CANTELEVER", "WHAT IS THE BEAM  
TYPE?"): BEAM = BEAM * 1
```

```
If BEAM = 1 Then GoTo 1000
```

```
If BEAM = 2 Then GoTo 2000
```

```

1000 Print " YOU ARE ANALYZING A SIMPLY SUPPORTED BOX CULVERT"
      REF = InputBox(" ENTER 1 FOR LEFT SUPPORT; 2 FOR RIGHT SUPPORT", "WHERE IS THE
REFERENCE POINT?"): REF = REF * 1
      If REF = 1 Then GoTo 1100
      If REF = 2 Then GoTo 1200

1100
1110 L = InputBox("WHAT IS THE LENGTH OF THE BEAM?")
      ALPHA = InputBox("WHAT IS POINT OF LOAD APPLICATION from left support?"): ALPHA =
ALPHA * 1
      Z = InputBox("WHAT IS THE POINT OF INTEREST ON THE BEAM?"): Z = Z * 1
      If Z > ALPHA Then MsgBox ("IT IS NOT ALLOWED! Z MUST BE LESS THAN ALPHA!!"): GoTo
1110

      BZ = B * Z: BAL = B * ALPHA
      SINHBZ = 0.5 * (ee ^ BZ - ee ^ -BZ)
      COSHBAL = 0.5 * (ee ^ BZ + ee ^ -BZ)
      SECHBAL = 1 / COSHBAL

      For X = 1 To NN
      PHY(X) = (MZ(X) / (B * G * J)) * (Z * B - SINHBZ / COSHBAL)
      PHY1(X) = (MZ(X) / (G * J)) * (1 - COSHBZ / COSHBAL)
      PHY11(X) = -(B * MZ(X)) / (G * J) * SINHBZ / COSHBAL
      PHY111(X) = -(B ^ 2 * MZ(X)) / (G * J) * COSHBZ / COSHBAL

      BiM(X) = -E * CW * PHY11(X)
      TW(X) = -E * SW * PHY111(X) / T
      WNS(X) = -E * WN * PHY11(X)

      PHYV(X) = MZ(X) * Z / (G * J)
      PD(X) = (PHY(X) - PHYV(X)) * 100 / PHYV(X)
      Next X
      GoTo 5000

1200
1210 L = InputBox("WHAT IS THE LENGTH OF THE BEAM?")
      ALPHA = InputBox("WHAT IS POINT OF LOAD APPLICATION from right support?"): ALPHA =
ALPHA * 1
      Z = InputBox("WHAT IS THE POINT OF INTEREST ON THE BEAM?")
      If Z < ALPHA Then MsgBox ("IT IS NOT ALLOWED! Z MUST BE GREATER THAN ALPHA!!"):
GoTo 1210

      BZ = B * Z: BAL = B * ALPHA: BL = B * L
      SINHBZ = 0.5 * (ee ^ (BZ) - ee ^ (-BZ))
      SINHBL = 0.5 * (ee ^ (BL) - ee ^ (-BL))
      SINHBAL = 0.5 * (ee ^ (BAL) - ee ^ (-BAL))
      COSHBZ = 0.5 * (ee ^ (BZ) + ee ^ (-BZ))
      COSHBL = 0.5 * (ee ^ (BL) + ee ^ (-BL))
      COSHBAL = 0.5 * (ee ^ (BAL) + ee ^ (-BAL))
      SECHBZ = 1 / COSHBZ
      SECHBAL = 1 / COSHBAL

```

```

SECHBL = 1 / COSHBL
TANHBZ = SINHBZ / COSHBZ
TANHBL = SINHBL / COSHBL
TANHBAL = SINHBAL / COSHBAL
COTHBZ = COSHBZ / SINHBZ
COTHBL = COSHBL / SINHBL
COTHBAL = COSHBAL / SINHBAL

```

```

For X = 1 To NN

```

```

  PHY(X) = MZ(X) / (B * G * J) * ((Z - L) + (SECHBAL * TANHBL * COSHBZ) / (1 - TANHBL *
COTHBAL) - SECHBAL * SINHBZ / (1 - TANHBL * COTHBAL))
  PHY1(X) = MZ(X) / (G * J) * (1 + SECHBAL * SINHBZ * TANHBL / (1 - TANHBL * COTHBAL) -
SECHBAL * COSHBZ / (1 - TANHBL * COTHBAL))
  PHY11(X) = B * MZ(X) / (G * J) * (SECHBAL * COSHBZ * TANHBL / (1 - TANHBL * COTHBAL) -
SECHBAL * SINHBZ / (1 - TANHBL * COTHBAL))
  PHY111(X) = B ^ 2 * MZ(X) / (G * J) * (SECHBAL * SINHBZ * TANHBL / (1 - TANHBL *
COTHBAL) - SECHBAL * COSHBZ / (1 - TANHBL * COTHBAL))

```

```

  BiM(X) = -E * CW * PHY11(X)
  TW(X) = -E * SW * PHY111(X) / T
  WNS(X) = -E * WN * PHY11(X)

```

```

  PHYV(X) = MZ(X) * Z / (G * J)
  PD(X) = (PHY(X) - PHYV(X)) * 100 / PHYV(X)
Next X

```

```

GoTo 5000

```

```

2000 Print " YOU ARE ANALYZING A CANTILEVER BOX CULVERT"

```

```

  REF = InputBox(" ENTER 1 FOR LEFT SUPPORT; 2 FOR RIGHT SUPPORT", "WHERE IS THE
REFERENCE POINT?"): REF = REF * 1

```

```

  If REF = 1 Then GoTo 2100

```

```

  If REF = 2 Then GoTo 2200

```

```

2100

```

```

2110 L = InputBox("WHAT IS THE LENGTH OF THE BEAM in M?")

```

```

  ALPHA = InputBox("WHAT IS POINT in M OF LOAD APPLICATION from left support?"): ALPHA =
ALPHA * 1

```

```

  Z = InputBox("WHAT IS THE POINT in M OF INTEREST ON THE BEAM?"): Z = Z * 1

```

```

  If Z > ALPHA Then MsgBox ("IT IS NOT ALLOWED! Z MUST BE LESS THAN ALPHA!!"): GoTo
2110

```

```

  BZ = B * Z: BAL = B * ALPHA: BL = B * L
  SINHBZ = 0.5 * (ee ^ (BZ) - ee ^ (-BZ))
  SINHBL = 0.5 * (ee ^ (BL) - ee ^ (-BL))
  SINHBAL = 0.5 * (ee ^ (BAL) - ee ^ (-BAL))
  COSHBZ = 0.5 * (ee ^ (BZ) + ee ^ (-BZ))
  COSHBL = 0.5 * (ee ^ (BL) + ee ^ (-BL))
  COSHBAL = 0.5 * (ee ^ (BAL) + ee ^ (-BAL))
  SECHBZ = 1 / COSHBZ
  SECHBAL = 1 / COSHBAL
  SECHBL = 1 / COSHBL
  COSECHBZ = 1 / SINHBZ

```

$\text{COSECHBAL} = 1 / \text{SINHBL}$   
 $\text{COSECHBL} = 1 / \text{SINHBL}$   
 $\text{TANHBZ} = \text{SINHBZ} / \text{COSHBL}$   
 $\text{TANHBL} = \text{SINHBL} / \text{COSHBL}$   
 $\text{TANHBAL} = \text{SINHBAL} / \text{COSHBAL}$   
 $\text{COTHBZ} = \text{COSHBL} / \text{SINHBZ}$   
 $\text{COTHBL} = \text{COSHBL} / \text{SINHBL}$   
 $\text{COTHBAL} = \text{COSHBAL} / \text{SINHBAL}$

For X = 1 To NN

$\text{PHY}(X) = \text{MZ}(X) / (\text{B} * \text{G} * \text{J}) * (\text{Z} * \text{B} - \text{SINHBZ} + \text{COSHBL}(\text{COTHBAL} - \text{COSECHBAL}) +$   
 $(\text{COSECHBAL} - \text{COTHBAL}))$

$\text{PHY1}(X) = \text{MZ}(X) / (\text{G} * \text{J}) * (1 - \text{COSHBL} + \text{SINHBZ}(\text{COTHBAL} - \text{COSECHBAL}))$

$\text{PHY11}(X) = \text{B} * \text{MZ}(X) / (\text{G} * \text{J}) * (\text{COSHBL}(\text{COTHBAL} - \text{COSECHBAL}) - \text{SINHBZ})$

$\text{PHY111}(X) = \text{B}^2 * \text{MZ}(X) / (\text{G} * \text{J}) * (\text{SINHBZ}(\text{COTHBAL} - \text{COSECHBAL}) - \text{COSHBL})$

$\text{BiM}(X) = -\text{E} * \text{CW} * \text{PHY11}(X)$

$\text{TW}(X) = -\text{E} * \text{SW} * \text{PHY111}(X) / \text{T}$

$\text{WNS}(X) = -\text{E} * \text{WN} * \text{PHY11}(X)$

$\text{PHYV}(X) = \text{MZ}(X) * \text{Z} / (\text{G} * \text{J})$

$\text{PD}(X) = (\text{PHY}(X) - \text{PHYV}(X)) * 100 / \text{PHYV}(X)$

Next X

GoTo 5000

2200

2210 L = InputBox("WHAT IS THE LENGTH OF THE BEAM in M?")

ALPHA = InputBox("WHAT IS POINT in M OF LOAD APPLICATION from right support?"): ALPHA =  
 ALPHA \* 1

Z = InputBox("WHAT IS THE POINT in M OF INTEREST ON THE BEAM?")

If Z < ALPHA Then MsgBox ("IT IS NOT ALLOWED! Z MUST BE GREATER THAN ALPHA!!");

GoTo 2210

$\text{BZ} = \text{B} * \text{Z}$ ;  $\text{BAL} = \text{B} * \text{ALPHA}$ ;  $\text{BL} = \text{B} * \text{L}$

$\text{SINHBZ} = 0.5 * (\text{ee}^{\text{BZ}} - \text{ee}^{-\text{BZ}})$

$\text{SINHBL} = 0.5 * (\text{ee}^{\text{BL}} - \text{ee}^{-\text{BL}})$

$\text{SINHBAL} = 0.5 * (\text{ee}^{\text{BAL}} - \text{ee}^{-\text{BAL}})$

$\text{COSHBL} = 0.5 * (\text{ee}^{\text{BL}} + \text{ee}^{-\text{BL}})$

$\text{COSHBL} = 0.5 * (\text{ee}^{\text{BL}} + \text{ee}^{-\text{BL}})$

$\text{COSHBAL} = 0.5 * (\text{ee}^{\text{BAL}} + \text{ee}^{-\text{BAL}})$

$\text{SECHBZ} = 1 / \text{COSHBL}$

$\text{SECHBAL} = 1 / \text{COSHBAL}$

$\text{SECHBL} = 1 / \text{COSHBL}$

$\text{COSECHBZ} = 1 / \text{SINHBZ}$

$\text{COSECHBAL} = 1 / \text{SINHBAL}$

$\text{COSECHBL} = 1 / \text{SINHBL}$

$\text{TANHBZ} = \text{SINHBZ} / \text{COSHBL}$

$\text{TANHBL} = \text{SINHBL} / \text{COSHBL}$

$\text{TANHBAL} = \text{SINHBAL} / \text{COSHBAL}$

$\text{COTHBZ} = \text{COSHBL} / \text{SINHBZ}$

$\text{COTHBL} = \text{COSHBL} / \text{SINHBL}$

$\text{COTHBAL} = \text{COSHBAL} / \text{SINHBAL}$

$$H1 = (\text{COTHBAL} - \text{COSECHBAL} - \text{TANHBAL}) / (\text{TANHBAL} - \text{TANHBL})$$

For X = 1 To NN

$$\text{PHY}(X) = \text{MZ}(X) / (\text{B} * \text{G} * \text{J}) * (\text{Z} * \text{B} + \text{H1} * \text{SINHBZ} - \text{H1} * \text{TANHBL} * \text{COSHBZ} + (\text{COSECHBAL} - \text{COTHBAL}))$$

$$\text{PHY1}(X) = \text{MZ}(X) / (\text{G} * \text{J}) * (1 + \text{H1} * \text{COSHBZ} - \text{H1} * \text{TANHBL} * \text{SINHBZ})$$

$$\text{PHY11}(X) = \text{B} * \text{MZ}(X) * \text{H1} / (\text{G} * \text{J}) * (\text{SINHBZ} - \text{TANHBL} * \text{COSHBZ})$$

$$\text{PHY111}(X) = \text{B}^2 * \text{MZ}(X) * \text{H1} / (\text{G} * \text{J}) * (\text{COSHBZ} - \text{TANHBL} * \text{SINBZ})$$

$$\text{BiM}(X) = -\text{E} * \text{CW} * \text{PHY11}(X)$$

$$\text{TW}(X) = -\text{E} * \text{SW} * \text{PHY111}(X) / \text{T}$$

$$\text{WNS}(X) = -\text{E} * \text{WN} * \text{PHY11}(X)$$

$$\text{PHYV}(X) = \text{MZ}(X) * \text{Z} / (\text{G} * \text{J})$$

$$\text{PD}(X) = (\text{PHY}(X) - \text{PHYV}(X)) * 100 / \text{PHYV}(X)$$

Next X

GoTo 5000

5000 PRINT RESULT

www = 1

Print "THANK YOU FOR USING THIS SOFTWARE": Print: Print

Print " SHEAR MODULUS = "; G; "N/M2"

Print " TORSIONAL CONSTANT = "; J; "M4"

Print " ELASTIC MODULUS = "; E; "N/M2"

Print " WARP CONSTANT = "; CW; "M6"

GG = InputBox("PRESS OK TO continue"): Cls

For X = 1 To NN

Print "ANGLE OF TWIST IS VLASOV"; X; " = "; PHY(X); " radians"

Print "ANGLE OF TWIST BASED ON SAINT VENANT IS"; X; " = "; PHYV(X); " radians": Print

Print "1ST DERIVATIVE OF ANGLE OF TWIST IS"; X; " = "; PHY1(X); " radians / M": Print

Print "2ND DERIVATIVE OF ANGLE OF TWIST IS"; X; " = "; PHY11(X); "radians / M2": Print ""

Print "3RD DERIVATIVE OF ANGLE OF TWIST IS"; X; " = "; PHY111(X); "radians / M3": Print

Print "PERCENTAGE DIFFERENCE "; X; " = "; PD(X); "%"

Print " Bi MOMENT "; X; " = "; BiM(X); " NM2"

Print " WARPING SHEAR STRESS "; X; " = "; TW(X); " N/M2"

Print " WARPING NORMAL STRESS "; X; " = "; WNS(X); "N/M2"

If www = 2 Then HHH = InputBox("PRESS OK to continue"): Cls: www = 0

www = www + 1

Next X

End Sub